A STUDY OF INFLUENCE OF VARIOUS FLOW PARAMETERS OF BILE IN HUMAN BILIARY SYSTEM

A Synopsis

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1 INTRODUCTION

Related Preliminaries

1.1 Bile

Bile (gall) is a greenish-yellow bio-fluid produced by the liver, that aids in the digestion of lipids in the small intestine. In humans, bile is produced continuously by the liver (liver bile), stored and concentrated in the gallbladder.

Components of Bile :

i. Water.
ii. Cholesterol.
iii. Lecithin (a phospholipid)
iv. Bile pigments (bilirubin & biliverdin)
v. Bile salts and bile acids (sodium glycocholate & sodium taurocholate)
vi. Small amounts of copper and other excreted metals [5].

1.2 Biliary System

Biliary system consists of liver, gallbladder and biliary ducts. Bile is the bio-fluid secereted by the hepatocytes (cells of the liver), which is responsible for the emulsification of fats. The biliary tract refers to the path by which bile is secreted by the liver and then transported to the duodenum.

Components of the biliary system:

There are mainly three components of the biliary system

1. Liver
2. Gallbladder
3. Biliary Ducts

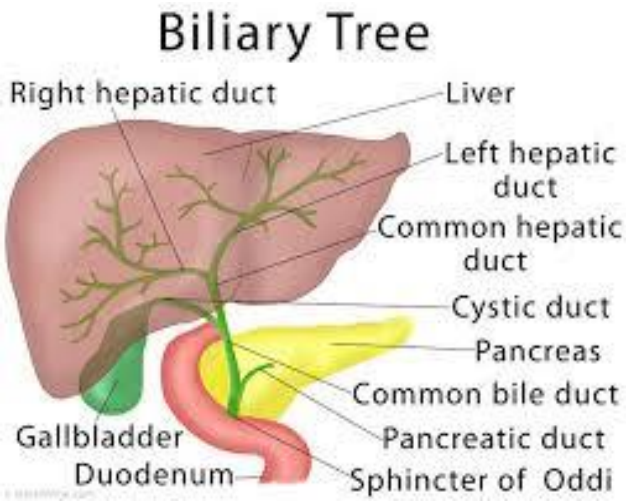

FIG. 1 ANATOMY OF BILIARY SYSTEM [4]

1.Liver

The liver is a reddish-brown, wedge-shaped organ with four lobes of unequal size and shape. Lobules are the functional units of the liver. Each lobule is made up of millions of hepatic cells (hepatocytes), which are the basic metabolic cells. These lobules are connected to small ducts (tubes) that connect with larger ducts to form the common hepatic duct. The common hepatic duct transports the bile made by the liver cells to the gallbladder and duodenum (the first part of the small intestine) via the common bile duct. Its other roles in metabolism include the regulation of glycogen storage, decomposition of red blood cells and the production of hormones [27].

2. Gallbladder

In vertebrates, the gallbladder is a small hollow organ where bile is stored and concentrated before it is released into the small intestine. In humans, the pear-shaped gallbladder lies beneath the liver, although the structure and position of the gallbladder can vary significantly among animal species. It receives and stores bile, produced by the liver, via the common hepatic duct and releases it via the common bile duct into the duodenum, where the bile helps in the digestion of fats [15].

3. Biliary Ducts

The bile ducts are the medium of transportation of bile from the liver and gallbladder through the pancreas to the duodenum, which is a part of the small intestine.
There are three types of the biliary duct:

a) Common Hepatic Duct - The common hepatic duct is the part of the biliary tract formed by the convergence of the right hepatic duct (which drains bile from the right functional lobe of the liver) and the left hepatic duct (which drains bile from the left functional lobe of the liver). The hepatic duct then joins the cystic duct coming from the gallbladder to form the common bile duct. The duct is usually 6–8 cm length [11].

b) Cystic Duct - The cystic duct connects the top of the gallbladder's neck to the common hepatic duct. Then it joins the common bile duct, which meets pancreatic duct before it empties into the duodenum. The cystic duct usually measures 2–4 cm in length and contains prominent concentric folds known as the spiral valves of Heister. The cystic duct frequently exhibits a tortuous or serpentine course. The normal diameter of the cystic duct is variable, ranging from 1 to 5 mm [12].

c) Common Bile Duct - The common bile duct is a small, tube-like structure formed where the common hepatic duct and the cystic duct join. Its physiological role is to carry bile from the gallbladder and empty it into the upper part of the small intestine (the duodenum) [10].

1.3 Biliary Diseases

Biliary disease refers to diseases affecting the bile ducts, gallbladder and other structures involved in the production and transportation of bile.
Most common biliary diseases are: [6]

1.3.1 Cholelithiasis

A gallstone is a stone formed within the gallbladder out of bile components. The term cholelithiasis may refer to the presence of gallstones or to the diseases caused by gallstones. Most gallstones (>90%) in developed countries consist of cholesterol. Cholesterol gallstone formation begins with the secretion of bile supersaturated with cholesterol from the liver. Initiated by nucleating factors such as mucin, microscopic crystals then precipitate in the gallbladder where hypomotility provides time for stone growth. Complications of gallstones may include inflammation of the gallbladder (cholecystitis), inflammation of the pancreas (pancreatitis), jaundice, and infection of a bile duct [9].

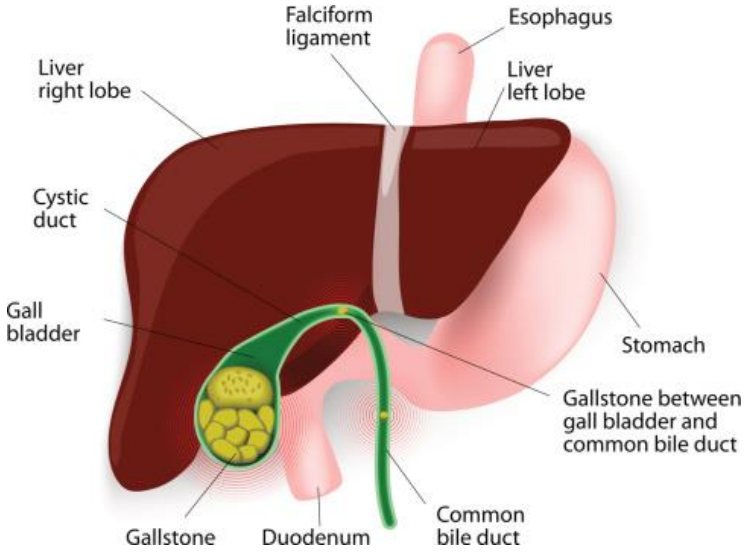

FIG 2 : GALLBLADDER WITH CHOLELITHIASIS [16]

1.3.2 Cholecystitis

Cholecystitis is an inflammation of the gallbladder, a condition that can turn deadly if left untreated. Cholecystitis occurs when the gallstones block the entrance of the gallbladder, resulting in the bile to build up and the gallbladder to become swollen. Most people are familiar with the gallbladder itself, but

aren’t sure exactly what it does or where it is located. When gallstones form and block the tube leading to the gallbladder, the patient is at risk of developing cholecystitis [8].

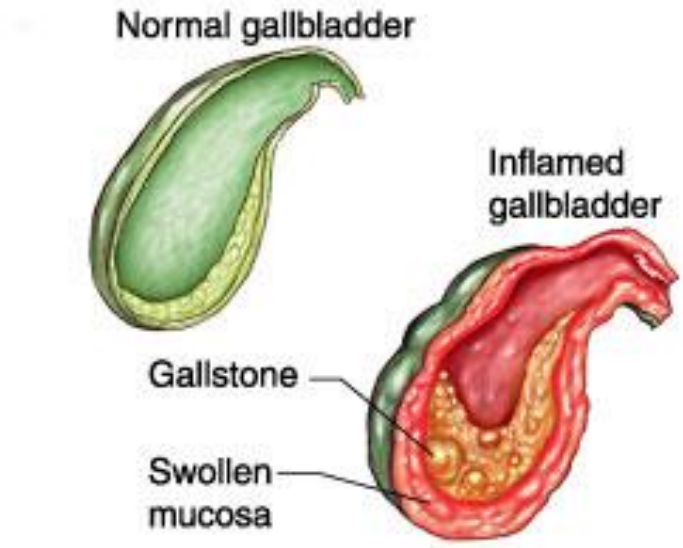

Fig 3 : NORMAL V/S INFLAMED GALLBLADDER [32]

1.3.3. Papillary Stenosis

Papillary stenosis is a disturbance of the sphincter of Oddi, a muscular valve, that prevents the opening and release of bile or pancreatic fluids into the duodenum in response to food entering the duodenum [35].

1.4 Viscosity of Fluid

Viscosity is a measure of a fluid's resistance to flow. It describes the internal friction of a moving fluid. A fluid with large viscosity resists motion because its molecular makeup gives it a lot of internal friction [38].

1.5 Newton's law of viscosity

A relation between the shear stress and the velocity of fluid was first stated by Newton, He, concluded that internal friction between two adjacent fluid particles should be independent of the normal pressure but proportional to the difference in their velocities.

Newton's law of viscosity

$$\tau \alpha \frac{du}{dy}$$

$$\tau = \mu \frac{du}{dy}$$

where,

μ = viscosity

τ = shear stress

$\frac{du}{dy}$ = rate of shear deformation [31].

1.6 Newtonian and Non-Newtonian Fluid

Fluids which obey Newton's Law of viscosity which says that shear stress is directly proportional to shear rate. Like water, air, mercury.

The constitutive equation for Newtonian fluids is

$\tau = \mu e$

where, τ is shear stress, e is shear rate and the ratio of shear stress and the shear rate is a constant which is termed as coefficient of viscosity denoted by μ.

The non-Newtonian fluids are those fluids which do not obey Newton's Law of viscosity. For these, ratio of shear stress to shear rate is not constant. Like paints, jam, polymer solution.

There are few important classes of non-Newtonian fluids which are

1. *Power Law Fluid*

   The constitutive equation is

   $$\tau = \mu e^n$$

   where n is the flow behaviour index.

   If $n < 1$ we get a pseudo-plastic power-law fluid and for $n > 1$ we get a dilatants power law fluid.

   This is found to be good for strain rates between 5 and 500 sec^{-1} with n having the value between 0.68 and 0.80.

2. *Bingham Plastic Fluid*

   The constitutive equations are

   $$\tau = \mu e + \tau_o \quad , \tau \geq \tau_o$$

   $$e = 0 \quad , \tau < \tau_o$$

   where τ_o is the yield stress.

3. *Herschel-Bulkley Fluid*

   The constitutive equations are

   $$\tau = \mu e^n + \tau_o \quad , \tau \geq \tau_o$$

   $$e = 0 \quad , \tau < \tau_o$$

   where n is the flow behaviour index.

   If $n < 1$ the fluid is shear thinning fluid, while $n > 1$ produces shear thickening fluid.

4. *Casson Fluid*

   The constitutive equations are

$\tau^{1/2} = \mu^{1/2}e^{1/2} + \tau_o^{1/2}$, $\tau \geq \tau_o$

$e = 0$, $\tau < \tau_o$

This holds the strain rate between 0 and 100,000 sec^{-1} [17].

5. *Carreau's Fluid*

The constitutive relation for the non-Newtonian fluid flow described by the carreau's equation

$$\bar{S}_{xy} = -p\delta_{xy} + \bar{\tau}_{xy}$$

$$\bar{\tau}_{xy} = \left[\eta_{xy} + (\eta_0 - \eta_\infty)(1 + (\dot{\gamma})^2)^{\frac{m-1}{2}}\right]\bar{\dot{\gamma}}_{xy}$$

Where p is the pressure, δ_{xy} is kronecker delta, $\bar{\tau}_{xy}$ is the extra stress tensor, η_0 is the zero shear rate viscosity, η_∞ is the infinite shear rate viscosity, $\bar{\dot{\gamma}}_{xy}$ is the time constant, m is the power index.

$\bar{\dot{\gamma}}$ is defind as follows:

$$\bar{\dot{\gamma}} = \sqrt{\frac{1}{2}\sum\sum \bar{\dot{\gamma}}_{xy}\bar{\dot{\gamma}}_{yx}} = \sqrt{\frac{1}{2}\pi}$$

Where π is the second invariant strain rate tensor. In case of $\eta_\infty = 0$ and applying Taylor's expansion, equation is converted into:

$$\bar{\tau}_{xy} = \eta_0\left[+\frac{m-1}{2}((\dot{\gamma})^2)\right]\bar{\dot{\gamma}}_{xy}$$ [7].

1.7 Peristaltic Motion

Peristalsis, involuntary movements of the longitudinal and circular muscles, primarily in the digestive tract but occasionally in other hollow tubes of the body, that occur in progressive wave like contractions. Peristaltic waves occur in the esophagus, stomach, and intestines. The waves can be short, local reflexes or long, continuous contractions that travel the whole length of the organ, depending upon their location and what initiates their action [36].

1.8 Porous Medium

A porous medium or a porous material is a material containing pores between solid materials through which liquid or gas can pass. Few examples of naturally occurring porous media are sand, soil and some types of stone such as pumice and sandstone. A porous medium can be distinguished from other porous media by its porosity (Measure of void space) or the size of its pores. Materials with low porosity are less permeable and have smaller pores, making it difficult for gas or liquid to pass through them, while materials with high porosity have large pores and are easily permeable.

1.9 Permeability

Permeability is the property of material that is an indication of the ability for fluids (gas or liquid) to flow through material. It only depends on the porosity of the material and independent of other properties of the material. High permeability will allow fluids to move rapidly through material.

2 Literature Review

Pitt et al. (1982) observed that resistance of cystic duct increases while sphincter of oddi resistance is unchanged in the presence of lithogenic bile without gallstones. They concluded that increased resistance to flow across the cystic duct, and not altered gallbladder compliance is etiological related to bile statis is an important event in gallstone formation [37].

Misra et al. (1995) developed a mathematical model for studying the peristaltic motion of newtonian fluid through a circular cylindrical tube of changing cross section, considering viscous and incompressible fluid. The problem is investigated by perturbation technique and analysis is based upon second order approximation [29].

Ooi et al. (2004) discussed the flow of bile in the human cystic duct. It is obtained that most significant geometric parameters affecting the resistance is the baffle clearance, followed by the number of baffles, while the least important are the angle between the duct and gallbladder [33].

Dang et al. (2004) developed the model for common bile duct and the remodeling data were expressed mathematically in terms of indicial response functions (IRF), i.e. change of particular feature of a common bile duct (CBD) in response to a unit step change of the pressure [13].

Al-Atabi et al. (2006) presented a mathematical model accounting for the effects of geometry of the baffle configurations. The model was solved algebraically for flow in pipe with three baffle arrangements and the results were validated by experimental data [2]. Further in (2008) presented an experimental study of flow of bile in complaint idealised human cyctic duct during both the filling and the emptying phases of the gallbladder by providing structural support and preventing the duct from total collapse [3].

Li et al. (2007) developed a model for the human biliary system during the emptying phase, based on a clinical test in which gallbladder volume changes were measured in response to a standard stimulus and a recorded pain profile. The peak total stress, including the passive and active stresses experienced by the gallbladder during the emptying [25]. In (2007) they also worked on one dimensional models to estimate the pressure drop in the normal human biliary system for Reynolds number up to 20. It was found that the maximum pressure drop occurs during bile emptying immediately after a meal, and was greatly influenced by the viscosity of bile and the geometric configuration of the cystic duct [26]. In (2008) they extended their previous study of the human biliary system to include two new factors: the non–Newtonian properties of bile, and elastic deformation of the cystic duct. A one – dimensional (1D) model was analyzed and compared with three–dimensional fluid structure interaction simulations. It was found that non – Newtonian bile raises resistance to the flow of bile, which can be increased enormously by the elastic deformation of the cystic duct [24].

Muthu et al. (2008) presented a paper for peristaltic motion of micropolar fluid in a circular cylindrical flexible tube with viscoelastic walls where fluid is considered as incompressible, unsteady and micropolar in an infinite tube and finite difference method is taken into account for solving the problem. They observed that radial velocity decreases when ratio of viscosity coefficient increases and magnetic parameter decreases [30].

Pandey et al. (2010) discussed peristaltic transport of a viscous, non-Newtonian, incompressible visco elastic fluid through a circular tube whose cross section is changing along the length and wall motion is governed by a wave of contraction propagating along the wall of the tube which contracts in the radial direction and then relaxes up to the boundary of the fully distended tube. Study reveals that flow rate decreases with relaxation time and increases with Reynolds number while pressure increases with relaxation time and decreases with Reynolds number at a given tapering [34].

Maiti et al. (2011) developed a mathematical model with an aim to study the peristaltic flow of bile through a porous channel within ducts in a pathological state. The velocity profile and the critical pressure for the occurance of reflux are investigated by using the numerical technique. It is seen that in the presence of gallstone, bile viscosity increases as the value of the porosity parameter increases, whereas the critical pressure for the reflux decreases as porosity increases [28].

Khan et al. (2013) studied the effects of variable viscosity on the peristaltic flow of non-Newtonian fluid through a porous medium in an inclined channel. The flow is investigated in a wave frame of reference moving with velocity of wave. It is noticed that peristaltic pumping region decreases with increasing the slip parameter and trapped bolus increases on increasing the porosity parameter and Deborah number [18].

Kuchumov et al. (2014) studies pathological human bile taken from the gallbladder and bile ducts and found that at higher shear stress the pathological bile behave like Newtonian fluid and also noted that the viscosity of the gallbladder bile is higher than that duct one. The maximum viscosity is achieved at initial shear stress and after then curves tend to reach saturation [20]. Kuchumov et al. also (2015) studied the modelling of peristaltic bile flow in the papilla ampoule with stone in the papillary stenosis and obtained the analytical solution for axial velocity, flow rate, and pressure. It is noticed that flow rate may have negative values corresponding to the case of reflux occur, the critical pressures were also computed. The pressure rise magnitudes corresponding to reflux occurring condition depending on various parameters were obtained [21]. They also (2017) presented peristaltic flow of lithogenic bile in the vateri's papilla as non-Newtonian fluid in finite length tube. Pressure distribution versus axial coordinate at different time instants are plotted for various values of Weissenberg number and amplitude ratio. It revealed that the

amplitude ratio has more effect on the pressure distribution along the tube compared to the Weissenberg number [22]. He further (2018) studied fluid motion in the elastic and compliant ducts and bile flow in extra hepatic ducts is simulated using FSI alogorithm also the geometry of biliary system was obtained using MRI patient-specific data. It was confirmed that normal bile can be modeled as Newtonian fluid and lithogenic bile can be modeled as non-Newtonian fluid (Carreau fluid) [19].

Agarwal et al. (2014) analyzed the flow of bile in the elastic cystic duct during the emptying phase. Bile behaves as a non-Newtonian fluid and power law fluid model is taken in account to study the flow of bile in the duct. It is noticed that the flow of bile increases with the increase in flow index parameter and with the increase in radius of the duct. However, with the increase in viscosity of gallbladder bile the flow of bile in the duct decreases [1].

Lachiheb (2014) investigated the effect of variability of viscosity on the peristaltic motion of incompressible Newtonian fluid through an axisymmetric tube and viscosity is assumed to be function of both radius and the axial coordinate. It is seen that variation of viscosity parameters affects the pressure rise, the friction force, pumping region and trapping limits but the reflux limit and free pumping are independent of variation of viscosity parameters [23].

Elmaboud et al. (2014) analyzed the effect of peristaltic motion of an incompressible Newtonian fluid with variable viscosity induced by periodic sinusoidal wave propagating along the wall of the finite length tube. . It has been shown that the peaks of pressure fluctuate with time and attain different values with non-integral number of peristaltic wave [14].

3 Purpose of study

This work is used to investigate the flow mechanics of bile flow through diseased duct or gallbladder of biliary system. The disease seems to originate due to deposition of cholesterol and stone inside the bile duct and gallbladder. Different mathematical models have been studied by some researchers to explore the various aspects of bile flow in diseased biliary system. Proposed mathematical models

comprehending the real world problem of flow of bile in the diseased biliary system will be formulated and analyzed.

4 Proposed work

In the proposed work some of the following types of problems will be addressed:

- **Study of effect of heat transfer on bile flow in tapered duct in the presence of papillary stenosis.**

  Governing equations of motion are

$$\frac{\partial \bar{U}}{\partial \bar{X}} + \frac{\partial \bar{V}}{\partial \bar{Y}} = 0$$

$$\rho\left(\frac{\partial}{\partial t} + \bar{U}\frac{\partial}{\partial \bar{X}} + \bar{V}\frac{\partial}{\partial \bar{Y}}\right)\bar{U} = -\frac{\partial \bar{p}}{\partial \bar{X}} + \frac{\partial}{\partial \bar{X}}\bar{S}_{XX} + \frac{\partial}{\partial \bar{Y}}\bar{S}_{XY} + \rho g \alpha_1 (T' - T_0)$$

$$\rho\left(\frac{\partial}{\partial t} + \bar{U}\frac{\partial}{\partial \bar{X}} + \bar{V}\frac{\partial}{\partial \bar{Y}}\right)\bar{V} = -\frac{\partial \bar{p}}{\partial \bar{Y}} + \frac{\partial}{\partial \bar{X}}\bar{S}_{XY} + \frac{\partial}{\partial \bar{Y}}\bar{S}_{YY}$$

$$\rho c_p\left(\frac{\partial \bar{T}}{\partial \bar{t}} + \bar{U}\frac{\partial \bar{T}}{\partial \bar{X}} + \bar{V}\frac{\partial \bar{T}}{\partial \bar{Y}}\right) = K\left(\frac{\partial^2 \bar{T}}{\partial \bar{X}^2} + \frac{\partial^2 \bar{T}}{\partial \bar{Y}^2}\right) + Q_0$$

  Where ρ is the density, $\bar{U}\ and\ \bar{V}$ are the velocity components, $\bar{p}$ is the pressure, $\bar{S}_{ij}$ is the tensor stress, T is the temperature, Q_0 is the constant heat addition / absorption, c_p is the specific heat at constant pressure, g is the acceleration due to gravity.

- **Problem on peristaltic lithogenic bile flow through the porous duct with slip condition.**

  Governing equations of motion are

$$\frac{1}{\bar{R}}\frac{\partial(\bar{R}\bar{U})}{\partial \bar{R}} + \frac{\partial \bar{W}}{\partial \bar{Z}} = 0$$

$$\rho\left(\frac{\partial \bar{W}}{\partial t} + \bar{W}\frac{\partial \bar{W}}{\partial \bar{Z}} + \bar{U}\frac{\partial \bar{W}}{\partial \bar{R}}\right) = -\frac{\partial \bar{p}}{\partial \bar{Z}} + \frac{1}{\bar{R}}\frac{\partial}{\partial \bar{R}}(\bar{R}\bar{S}_{13}) + \frac{\partial}{\partial \bar{Z}}\bar{S}_{33} - \frac{\mu}{k}\bar{W}$$

$$\rho\left(\frac{\partial \bar{U}}{\partial t}+\bar{W}\frac{\partial \bar{U}}{\partial \bar{Z}}+\bar{U}\frac{\partial \bar{U}}{\partial \bar{R}}\right)=-\frac{\partial \bar{p}}{\partial \bar{R}}+\frac{1}{\bar{R}}\frac{\partial}{\partial \bar{R}}(\bar{R}\bar{S}_{11})+\frac{\partial}{\partial \bar{Z}}\bar{S}_{31}-\frac{\mu}{k}\bar{U}$$

Where $\bar{W}\ and\ \bar{U}$ are the velocity components, $\bar{p}$ is the pressure, $\bar{S}_{ij}, i,j=1,3$ are the extra tensor stress, ρ is the density, μ is viscosity and k is the porosity.

- **Study of peristaltic flow of bile in a porous channel.**

  Governing equations of motion are

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0$$

$$\rho\left(\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}\right)=-\frac{\partial p}{\partial x}+\mu_1\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}\right)-\frac{\mu}{k}u$$

$$\rho\left(\frac{\partial u}{\partial t}+u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}\right)=-\frac{\partial p}{\partial y}+\mu_1\left(\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}\right)-\frac{\mu}{k}v$$

  Where ρ is the density, $u\ and\ v$ are the velocity components, p is the pressure and $\mu_1=\frac{\mu}{e}$, μ is viscosity and e, k are respectively the porosity, permeability parameters.

- **Study of effect of radial variability of viscosity on the peristaltic transport of bile**.

  Governing equations of motion are

$$\frac{1}{r}\frac{\partial(\bar{r}\bar{u})}{\partial \bar{r}}+\frac{\partial \bar{w}}{\partial \bar{z}}=0$$

$$\frac{\partial \bar{p}}{\partial \bar{r}}=\frac{\partial}{\partial \bar{z}}\left[\bar{\mu}(r)\left(\frac{\partial \bar{u}}{\partial \bar{z}}-\frac{\partial \bar{w}}{\partial \bar{r}}\right)\right]$$

$$\frac{\partial \bar{p}}{\partial \bar{z}}=\frac{1}{\bar{r}}\frac{\partial}{\partial \bar{r}}\left[\bar{\mu}(r)\bar{r}\left(\frac{\partial \bar{u}}{\partial \bar{z}}-\frac{\partial \bar{w}}{\partial \bar{r}}\right)\right]$$

  Where $\bar{w}\ and\ \bar{u}$ are the velocity components in radial co-ordinates, $\bar{p}$ is the pressure and $\bar{\mu}(r)$ is viscosity which is function of r.

- **Study of influence of slip wall properties on the peristaltic transport of bile fluid through a inclined duct.**

  Governing equations of motion are

$$\frac{\partial \bar{U}}{\partial \bar{X}} + \frac{\partial \bar{V}}{\partial \bar{Y}} = 0$$

$$\rho\left(\frac{\partial \bar{U}}{\partial t} + \bar{U}\frac{\partial \bar{U}}{\partial \bar{X}} + \bar{V}\frac{\partial \bar{U}}{\partial \bar{Y}}\right) = -\frac{\partial \bar{p}}{\partial \bar{X}} + \frac{\partial \bar{\tau}_{XX}}{\partial \bar{X}} + \frac{\partial \bar{\tau}_{XY}}{\partial \bar{Y}} + \rho g \sin\alpha$$

$$\rho\left(\frac{\partial \bar{V}}{\partial t} + \bar{U}\frac{\partial \bar{V}}{\partial \bar{X}} + \bar{V}\frac{\partial \bar{V}}{\partial \bar{Y}}\right) = -\frac{\partial \bar{p}}{\partial \bar{Y}} + \frac{\partial \bar{\tau}_{XY}}{\partial \bar{X}} + \frac{\partial \bar{\tau}_{YY}}{\partial \bar{Y}} - \rho g \cos\alpha$$

  Where ρ is the density, $\bar{U}$ *and* $\bar{V}$ are the velocity components, $\bar{p}$ is the pressure, $\bar{\tau}_{ij}$ are the extra tensor stress, g is the acceleration due to gravity, α is the angle of inclination.

Methodology

Proposed models will be solved computationally and analytically using initial conditions and boundary conditions respectively to get the expression for axial velocity, pressure gradient, stream function and flow rate and the behaviour of these flow parameters will be analyzed graphically.

Expected Outcomes

In the proposed research work the behaviour of various flow parameters of bile such as axial velocity, pressure gradient, stream function, flow rate will be studied by analytical and computational methodology. Proposed mathematical models are designed in such way that they will produce impact of papillary stenosis, stenosis, gallstone on the resistance to flow, axial velocity, pressure gradient, stream function and flow rate.

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